Name ____ Test 2, Fall 2020

1) Find the determinant of the matrix below. (15 points)

$$\begin{bmatrix} 2 & 0 & 5 & 0 & 4 \\ 1 & 2 & 3 & 0 & 0 \\ -2 & 0 & 0 & 4 & 0 \\ 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$$

$$\begin{vmatrix} 2 & 0 & 5 & 0 & 4 \\ 1 & 2 & 3 & 0 & 0 \\ -2 & 0 & 0 & 4 & 0 \\ 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 5 \end{vmatrix} = 2\begin{vmatrix} 2 & 5 & 0 & 4 \\ -2 & 0 & 4 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{vmatrix} = 2\left(-1\begin{vmatrix} 2 & 5 & 4 \\ -2 & 0 & 0 \\ 1 & 2 & 0 \end{vmatrix} + 5\begin{vmatrix} 2 & 5 & 0 \\ -2 & 0 & 4 \\ 1 & 2 & 0 \end{vmatrix}\right)$$

$$= 2\left(-1 \cdot 4 \cdot \begin{vmatrix} -2 & 0 \\ 1 & 2 \end{vmatrix} + 5 \cdot (-4) \cdot \begin{vmatrix} 2 & 5 \\ 1 & 2 \end{vmatrix}\right)$$

$$= 2\big(-4(-4) - 20(4-5)\big)$$

$$= 2(16 + 20)$$

$$= 2(36) = 72$$

2) Given the basis and vector $\vec{x}_B$ below, find $\vec{x}_S$. (10 points)

$$B = \left\{\begin{bmatrix}3\\6\end{bmatrix}, \begin{bmatrix}2\\4\end{bmatrix}\right\} \quad \vec{x}_B = \begin{bmatrix}-1\\5\end{bmatrix}_B$$

$$-\begin{bmatrix}3\\6\end{bmatrix} + 5\begin{bmatrix}2\\4\end{bmatrix} = \begin{bmatrix}7\\14\end{bmatrix}$$

3) Given the two bases below, find the change of basis matrix that converts information from coordinate vectors in B_2 to coordinate vectors in B_1, denoted by $[I]_{B_1}^{B_2}$. You do not need to perform the arithmetic. (10 points)

$$B_1 = \left\{\begin{bmatrix}6\\0\\0\end{bmatrix}, \begin{bmatrix}0\\5\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}\right\} \quad B_2 = \left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\3\end{bmatrix}\right\}$$

$$\begin{bmatrix}1 & 0 & 1\\1 & 1 & 2\\0 & 1 & 3\end{bmatrix}^{-1}\begin{bmatrix}6 & 0 & 0\\0 & 5 & 1\\0 & 0 & 1\end{bmatrix}$$

4) Answer the questions below. (3 points each)

(A) Let A be a 3×3 matrix and $\vec{b} \in \mathbb{R}^3$. Assume the null space has dimension 1. How many solutions can $A\vec{x} = \vec{b}$ have?

0 or ∞

(B) Let A be a 7×4 matrix and assume $A\vec{x} = \vec{0}$ has only 1 solution. What is the rank of A?

4

(C) Let A be a 5×8 matrix whose corresponding linear transformation is onto. How many free variables does the system of equations $A\vec{x} = \vec{0}$ have?

3

(D) Let A be a 5×5 matrix with $|A| = 3$. How many solutions can $A\vec{x} = \vec{0}$ have?

1

(E) Let A be a 4×3 matrix with rank 2. What is the dimension of the row space of A?

2

5) Given the linear transformation below, determine whether or not it is *one-to-one* and justify your answer. (10 points. 3 for the answer; 7 for the reasoning)

$$T: \mathbb{R}^3 \to \mathbb{R}^3$$
$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

It is not one to one, because there is a column without a pivot. [which means that multiple different choices for the inputs x_1, x_2, and x_3 yield the same output)

6) Find the determinant of the matrix below. (5 points)

$$\begin{bmatrix} 1 & 4 \\ 5 & 2 \end{bmatrix}$$

$1 \cdot 2 - 5 \cdot 4 = -18$

7) Given the system of equations below, use Cramer's Rule to write down a formula for the solution. You do not need to simplify or evaluate your answer(s). (10 points)

$$\begin{bmatrix} 7 & 5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$x = \frac{\begin{vmatrix} 4 & 5 \\ 6 & 3 \end{vmatrix}}{\begin{vmatrix} 7 & 5 \\ 2 & 3 \end{vmatrix}} \quad y = \frac{\begin{vmatrix} 7 & 4 \\ 2 & 6 \end{vmatrix}}{\begin{vmatrix} 7 & 5 \\ 2 & 3 \end{vmatrix}}$$

8) Find the product below. (10 points)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 0 & -1 \end{bmatrix}$$

9) Find the inverse of the matrix below. (10 points)

$$\begin{bmatrix} 1 & 4 \\ 5 & 2 \end{bmatrix}$$

$$\frac{1}{-18}\begin{bmatrix} 2 & -4 \\ -5 & 1 \end{bmatrix}$$

10) Given the matrix below, find the quadratic form that corresponds to this matrix. (5 points)

$$\begin{bmatrix} 2 & 3 \\ 3 & 7 \end{bmatrix}$$

$$f(x, y) = 2x^2 + 6xy + 7y^2$$